Printed Page:- 04 **Subject Code:- AAS0301B**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: III - THEORY EXAMINATION (2023 - 2024)

Subject: Engineering Mathematics-III

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* ***three Sections -A, B, & C.*** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION-A 20

1. Attempt all parts:-

1-a. Bilinear Transformation preserve the cross ratio of _____ points. (CO1) 1

(a) 1

(b) 3

(c) 4

(d) None of these

1-b. The function f(z) = $\bar{z}$ is (CO1) 1

(a)
Analytic everywhere

(b)
Nowhere analytic

(c)
Analytic at origin

(d)
None of these

1-c. The region of validity for Taylor's series about $z = 0$ of the function e^z is (CO2) 1

(a) $|z| = 0$

(b) $|z| < 1$

(c) $|z| > 1$

(d) $|z| < \infty$

1-d. 1

If C is closed curve $|z| = r$ and $n \neq -1$ then the value of $\int_C z^n dz$ is (CO2)

(a) $2\pi i$

(b) 0

(c) πi

(d) $-2\pi i$

1-e. Condition for a PDE to be parabolic is (CO3) 1

(a) $B^2-4AC=0$

(b) $B^2-4AC>0$

(c) $B^2-4AC<0$

(d) None of these

1-f. 1

What is the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ when the ratio (k) is negative? (CO3)

(a) $u = (c_1 e^{px} + c_2 e^{-px}) c_3 e^{p^2 c^2 t}$

(b) $u = (c_1 x + c_2) c_3$

(c) $u = (c_1 Cospx + c_2 Sinpx) c_3 e^{-p^2 c^2 t}$

(d) *None of these*

1-g. 1

The Fourier transform of $F(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ (CO4)

(a) $\frac{\sin p}{p}$

(b) $\frac{2 \sin p}{p}$

(c) $\frac{\sin p}{2p}$

(d) None of these

1-h. If $Z\{f(k)\} = F(z)$ then $Z\{k^n f(k)\}$ is (CO4) 1

(a) $\left(z\frac{d}{dz}\right)^n F(z)$

(b) $\left(z\frac{d}{dz}\right)^2 F(z)$

(c) $\left(-z\frac{d}{dz}\right)^n F(z)$

(d) None of these

1-i. Raj swims 26 km downstream in same time as 14 km upstream. What is his speed 1

in still water if speed of stream is 3 km/hr? (CO5)

(a) 10 km/hr
(b) 12 km/hr
(c) 7 km/hr
(d) None of these

1-j. If Ram and Shyam together can build a house in 10 days; Ram and Arun can build it together in 12 days and Shyam and Arun can build it in 15 days. Shyam, Ram and Arun start working together. In how many days they build the house? (CO5) 1

(a) 6 days
(b) 4 days
(c) 8 days
(d) None of these

2. Attempt all parts:-

2.a. Define Harmonic Functions (CO1) 2

2.b. Evaluate: $\oint_C \frac{e^{-z}}{z} dz \; ; \; C \equiv |z| = 1.$ (CO2) 2

2.c. Find the c.f. of $(D^2 - D'^2)z = \cos(x+y)$.(CO3) 2

2.d. State the Fourier convolution theorem of two functions f(z) and g(z). (CO4) 2

2.e. Two pipes A and B can fill a tank in 20 and 30 minutes respectively. If both the pipes are used together, how long will it take to fill the tank? (CO5) 2

SECTION-B 30

3. Answer any five of the following:-

3-a. Show that the function defined by $f(z) = \sqrt{|xy|}$ is not regular at the origin although C-R equation is satisfied at origin. (CO1) 6

3-b. Check whether function $f(z) = 3x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic or not ?(CO1) 6

3-c. Evaluate $\oint_C \frac{3z^2+2}{(z-1)(z^2+9)} dz \; ; \; C \equiv |z-2| = 2.$ (CO2) 6

3-d. Evaluate $\oint_C \frac{1}{(z^2-4)^2} dz \; ; \; C \equiv |z| = 1.$ (CO2) 6

3.e. Solve the PDE: $(D^2 - a^2D'^2)z = x$ (CO3). 6

3.f. Find the inverse $Z-$transform of $F(z) = \frac{z}{z^2+7z+10}$. (CO4) 6

3.g. A thief is noticed by a policeman from a distance of 200 m. The thief starts running and the policeman chases him. The thief and the policeman run at the speed of 10km/hr and 11 km/hr respectively. What is the distance between them after 6 minutes? (CO5) 6

SECTION-C 50

4. Answer any one of the following:-

4-a. Find the bilinear transformation which maps the points $z = 1, -i, -1$ into the points $w = i, 0, -i$ respectively. (CO1) 10

4-b. If $u = e^x(x\cos y - y\sin y)$ is a harmonic function, find an analytic function f(z)=u+iv such that f(1)=e. (CO1) 10

5. Answer any one of the following:-

5-a. Determine the poles of the following function and residues at each poles $f(z) = \frac{z-1}{(z+1)^2(z-2)}$ and hence evaluate $\int_C f(z)\,dz$ where C is the circle $|z-i| = 2$. (CO2) 10

5-b. Expand $f(z) = \frac{1}{z^2-3z+2}$ 10

(i) $|z| < 1$

(ii) $1 < |z| < 2$

(iii) $0 < |z-1| < 1$

(iv) $|z| > 2$ (CO2)

6. Answer any one of the following:-

6-a. Solve the PDE: $(D^2 + DD' - 6D'^2)z = y\sin x$.(CO3) 10

6-b. Solve the PDE: $(D^2 + D'^2 - 2DD')z = e^{y+2x}$ (CO3). 10

7. Answer any one of the following:-

7-a. Find the inverse Z transform of function $F(z) = \frac{5}{(z-2)(z-3)}$ (CO4) 10

7-b. Find the fourier transform of e^{-x^2}. Hence find the Fourier transform of $e^{-\frac{x^2}{2}}$. (CO4) 10

8. Answer any one of the following:-

8-a. (i) X and Y can do a piece of work in 20 days and 12 days respectively. X started the work alone and then after 4 days Y joined him till the completion of the work. How long did the work last? 10

(ii) A man and a boy can do a piece of work in 24 days. If the man works alone for the last 6 days, it is completed in 26 days. How long would the boy take to do it alone? (CO5)

8-b. (i) A, B, C, D and E are sitting on a bench. A is sitting next to B, C is sitting next to D, D is not sitting with E who is on the left end of the bench. C is on the second position from the right. A is to the right of B and E. A and C are sitting together. In which position A is sitting? 10

(ii) 6 Boys are sitting in a circle and facing towards the Centre of the circle. Rajeev is sitting to the right of Mohan but he is not just at the left of Vijay. Suresh is between Babu and Vijay. Ajay is sitting to the left of Vijay. Who is sitting to the left of Mohan? (CO5)